Tutorial On Impact of Transformer Tap Changing On AC Power Flow

Concept of the Power Triangle

The power triangle is a useful tool in illustrating how MW & MVar combine to form the total power flow or MVA. Figure 1 illustrates the Power Triangle & states basic formulas for MVA, MW, MVar & power factor. These formulas are used in the following tutorial.

$$\mathbf{MVA^2 = MW^2 + jMVar^2}$$

$$\mathbf{MVA = \sqrt{MW^2 + jMVar^2}}$$

$$\mathbf{Power\ Factor = Cos\ \theta = \frac{MW}{MVA}}$$

$$\mathbf{MW = MVA \times Cos\ \theta}$$

$$\mathbf{MVar = MVA \times Sin\ \theta}$$

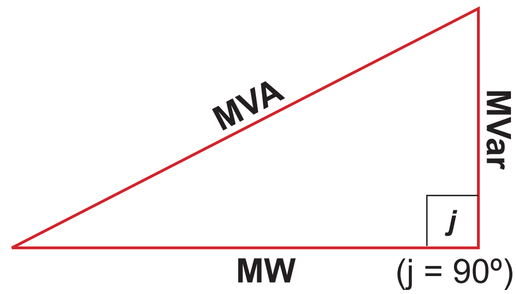


Figure 1. Power Triangle

Formulas For AC MW & MVar Flow

To transmit MW from point A to point B, the voltage phase angle at point A must lead the voltage phase angle at point B. The amount of MW transferred between points A & B is mainly a function of the voltage phase angle difference between the 2 points. Figure 2 illustrates the concept of MW flow & states the MW transfer equation. Power system resistance is ignored in this equation. If the effects of resistance were included, the complexity of the equation increases substantially. High voltage power systems normally have high inductive reactance (X) with low resistance so ignoring the resistance does not introduce much error.

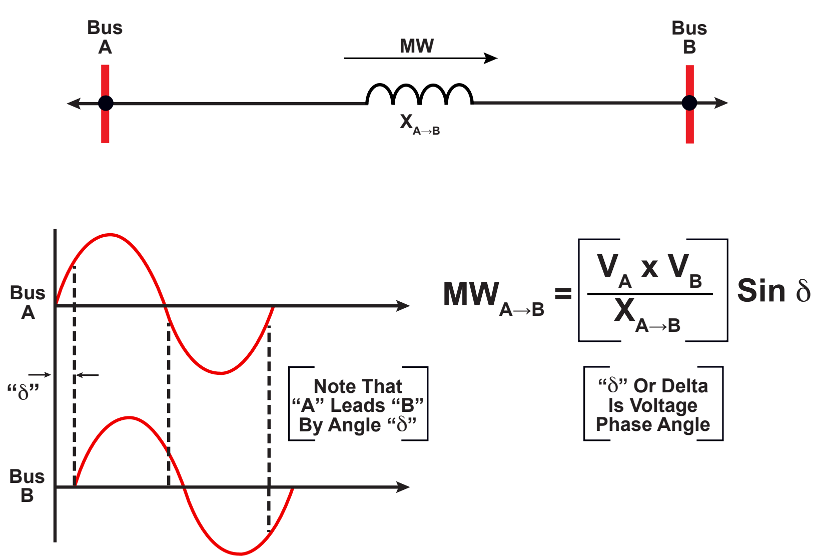
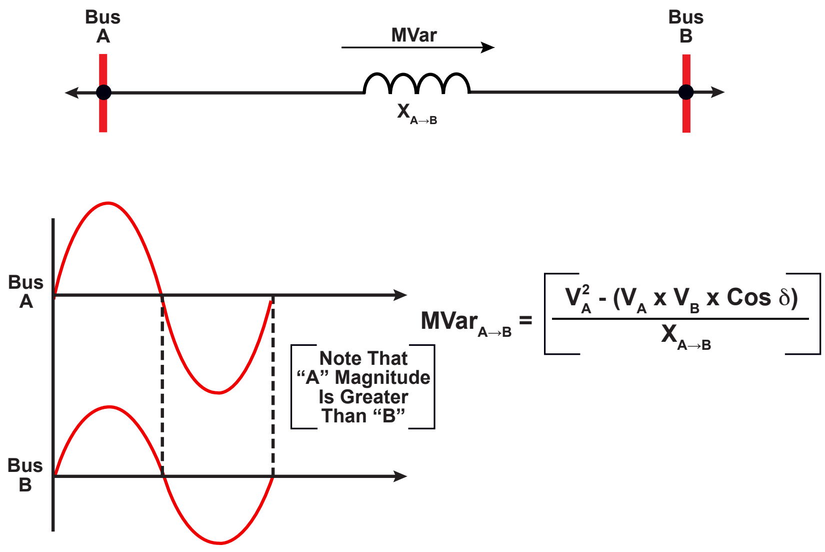


Figure 2. Illustration of MW Flow

To transmit MVar from point A to point B, the voltage magnitude at point A is normally greater than the voltage magnitude at point B. The voltage phase angle difference typically plays a less significant role in MVar flow than in MW flow. Figure 3 illustrates the concept of MVar flow & states the MVar transfer equation. In the same manner as the MW transfer equation, the system resistance is ignored. In addition, the power system's natural capacitance is ignored.



$$MVar_{A \to B} = \left[\frac{V_A^2 - (V_A \times V_B \times \cos \delta)}{X_{A \to B}} \right]$$

Figure 3. Illustration of MVar flow

In summary, MW flow is mainly based on the voltage phase angle difference while MVar flow is mainly based on the voltage magnitude difference. MW & MVar may flow in opposite directions on power lines. MW flow is downhill on angle while MVar flow is downhill on magnitude.

Illustration of AC Power Flow In Simple Power System

To illustrate AC power flow concepts, a power flow simulation was created using 3 identical 138/13.6 kV transformers to supply load as illustrated in Figure 4.

Explanation of Figure 4 power system details:

1. Customer load is 30 MW with a power factor of 0.91. Therefore the MVA = 30/0.91 = 33 & the $MVar^2 = (33^2 – 30^2)$ so MVar = 13.7
2. 138 kV & 13.6 kV are the base voltages for the power system studied. As illustrated in Figure 4, actual voltages for this system are equal to the base voltage of 138 kV times the per-unit value of 1.029 = 142 kV. Actual voltage on the 13.6 kV side is 13.6 x 1.043 = 14.2 kV.
3. 138 kV buses A1, A2 & A3 each supply 10 MW & 5 MVar to the high-side of their transformers. All transformers have the same impedance (0.35 per-unit) and all 3 transmission feeders have the same source in this power system.

4. The difference between the 15 MVar supplied to the high side of the 3 transformers & the load's reactive power usage of 13.8 MVar (1.2 MVar) is due to reactive power usage within the 3 transformers.
5. The 13.6 kV load bus is used as the reference bus & has a voltage phase angle of 0º. The voltage phase angles for 138 kV buses A1 – A3 are leading the load bus by 1.92º.

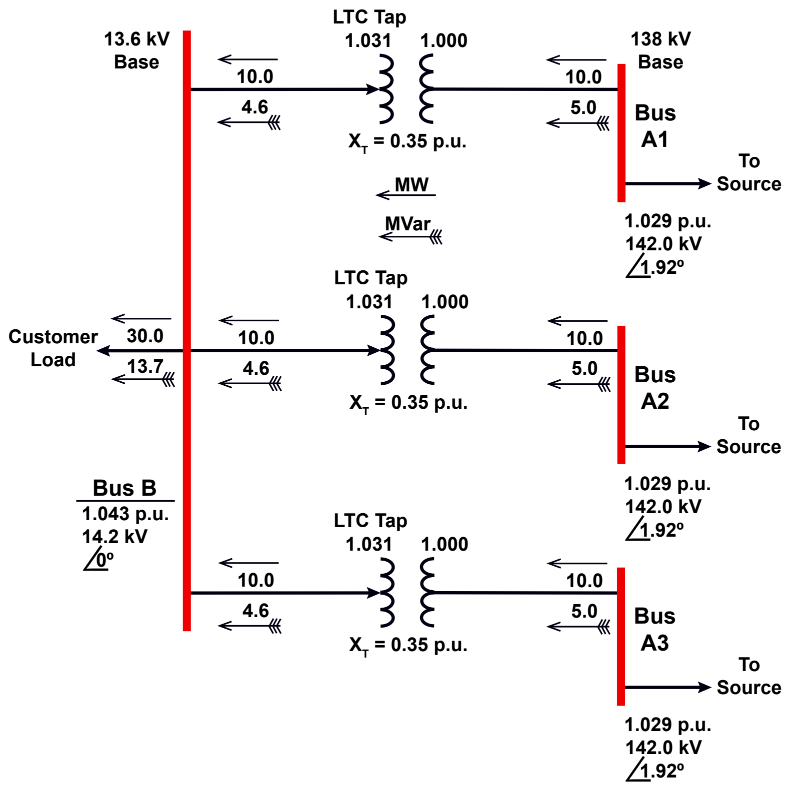


Figure 4. 138 kV to 13.6 kV Power System

Application of MW Transfer Equation

The MW flow from each of the 3 Bus A circuits is equal to:

$$\mathrm{MW}_{\mathrm{A}\rightarrow\mathrm{B}} = \frac{\mathrm{V}_A \times \mathrm{V}_B}{X_{A\rightarrow B}} \times \mathrm{Sin}\ \delta$$

Since the calculation is done using the per-unit system, the per-unit power must be multiplied by 100 (base MVA) to convert per-unit to MW. Using the actual system voltage, impedance & angle data the MW flow on each feeder is:

$$\mathrm{MW}_{\mathrm{A}\rightarrow\mathrm{B}} = \frac{1.029 \times 1.043}{0.35} \times \mathrm{Sin}\ 1.92° \times 100 = 10.3\ \mathrm{MW}$$

Note that the 10.3 MW calculated is very close to the 10.0 MW flow illustrated in the power flow data of Figure 4.

Application of MVar Transfer Equation

The MVar flow from each of the 3 Bus A circuits is equal to:

$$\mathrm{MVar}_{\mathrm{A\rightarrow B}} = \frac{V_A^2 - (\mathrm{V}_A \times V_B \text{ x} \cos \delta)}{X_{A\rightarrow B}}$$

Again, since the calculation is done using the per-unit system, the per-unit power is multiplied by 100 (base MVA) to convert per-unit to MVar. Using the actual system voltage, impedance & angle data the MVar flow is:

$$\mathrm{MVar}_{\mathrm{A\rightarrow B}} = \frac{1.029^2 - (1.029 \times 1.043 \text{ x} \cos 1.92°)}{0.35} \times 100 = -3.9 \text{ MVar}$$

The result of the MVar equation application is -3.9 MVar. The minus sign means the reactive power is flowing from Bus B to Bus A. These results do not agree with the 5.0 MVar from Bus A to Bus B that is illustrated in the power flows of Figure 4.

MVar flow is mainly a function of voltage magnitude. To transmit MVar from Bus A to Bus B, the voltage at Bus A should be greater than Bus B. As noted from the voltage values in Figure 4, Bus A (142 kV) is greater than Bus A (14.2 kV) so the minus sign on the calculated MVar flow appears incorrect. However, this MVar flow calculation is incorrect because a key factor was ignored in this calculation. The key factor is the role of the transformer LTC with respect to MVar flow.

The power transfer equations used in this tutorial are commonly seen in many reference books on power systems. These equations work fine as long as the power flows are calculated between 2 busses with nothing in between. However, the presence of the transformer & the LTC position are key factors that cannot be ignored in the equation application.

Modified Power Flow Equations With LTC Impact

Voltage magnitude has a large impact on MVar flow & ignoring the transformer tap position can create a large error in the MVar flow calculation. The MVar transfer equation is modified as shown below to include the impact of the transformer's LTC tap.

$$\mathrm{MVar}_{\mathrm{A\rightarrow B}} = \frac{\mathrm{V_A^2} - (\mathrm{V_A} \times \frac{\mathrm{V_B}}{\mathrm{LTC_{Tap}}} \text{ x} \cos \delta)}{\mathrm{X_{A\rightarrow B}}}$$

This modified MVar equation is used to re-calculate the MVar flow:

$$\mathrm{MVar}_{\mathrm{A\rightarrow B}} = \frac{1.029^2 - \left(1.029 \times \frac{1.043}{1.031} \text{ x} \cos 1.92°\right)}{0.35} \times 100 = 5.3 \text{ MVar}$$

Note that the 5.3 MVar calculated is very close to the 5.0 MVar flow illustrated in the power flow data of Figure 4. In addition, the flow is now from Bus A to Bus B which is also in agreement with the power flow data of Figure 4.

The MW flow is also impacted by the LTC position. The corrected equation is:

$$\mathrm{MW}_{\mathrm{A}\to\mathrm{B}} = \frac{V_A \times \dfrac{\mathrm{V_B}}{\mathrm{LTC_{Tap}}}}{X_{A\to B}} \times \mathrm{Sin}\ \delta$$

This modified MW equation is used to calculate the MW flow:

$$\mathrm{MW}_{\mathrm{A}\to\mathrm{B}} = \frac{1.029 \times \dfrac{1.043}{1.031}}{0.35} \times \mathrm{Sin}\ 1.92° \times 100 = 9.96\ \mathrm{MW}$$

Note that the 9.96 MW flow calculated is not that large of a change from the 10.3 MW calculated when the LTC tap was ignored. MW flow is more dependent on the angle than the voltage. The adjustment of the MVar equation for LTC inclusion had a much bigger impact on the MVar flow.

Additional Example Of Power Transfer Equation Usage

To further illustrate the usage of the MW & MVar transfer equations, modifications were made to the sample power system of Figure 4 to create Figure 5. The power system in Figure 5 has lost one of its transformers & a 10 MVar capacitor has been inserted on the 13.6 kV end of the system.

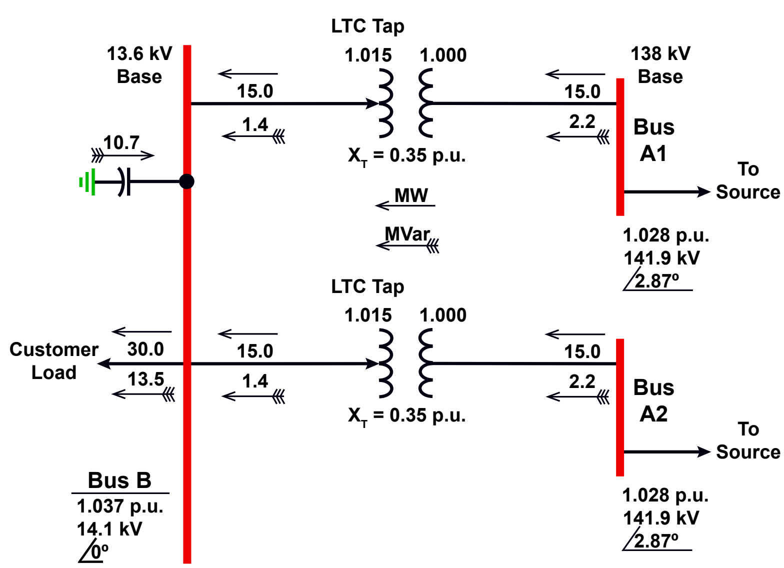


Figure 5. Transformer Outage & 10 MVar Capacitor In-Service

Explanation of Figure 5 power system details:

1. The 10 MVar capacitor bank is producing 10.7 MVar. The MVar production of a shunt capacitor is a function of the square of the voltage at which the capacitor operates. This capacitor would produce 10 MVar if it was operated at 13.6 kV. Because the capacitor is actually operating at 14.1 kV its actual MVar output is:
$$\mathrm{MVar_{Actual}} = \mathrm{MVar_{Rated}} \times \left[\frac{\mathrm{V_{Actual}}}{\mathrm{V_{Rated}}}\right]^2 = 10 \times \left[\frac{14.1}{13.6}\right]^2 = 10.7\ \mathrm{MVar}$$
2. Since the capacitor is supplying 10.7 MVar, the 2 transformers only have to supply 2.8 MVar to the load.
3. With transformer 3 out-of-service, the remaining 2 transformers supply the entire customer load. The power system now has a higher impedance since one of the supply paths is out-of-service. The higher impedance system now requires a higher voltage phase angle to move the MW. Note the voltage phase angle has increased to 2.87º compared to 1.92º in the Figure 4 power system.
4. The LTC tap is listed in Figure 5 as 1.015. This tap is used in the power transfer equations to calculate the MW & MVar flows:

$$\mathrm{MW_{A\to B}} = \frac{1.028 \times \frac{1.037}{1.015}}{0.35} \times \mathrm{Sin}\ 2.87° \times 100 = 15.0\ \mathrm{MW}$$

$$\mathrm{MVar_{A\to B}} = \frac{1.028^2 - \left(1.028 \times \frac{1.037}{1.015}\ \mathrm{x\,Cos}\ 2.87\,°\right)}{0.35} \times 100 = 2.2\ \mathrm{MVar}$$

Both the MW & MVar flows match the power flows given in Figure 5. The two rules of thumb provided earlier in this tutorial can be modified to reflect what we have learned.

- MW Flows Downhill on Angle
- MVar Flows Downhill on Per-Unit Voltage

The above tutorial was inspired by an article posted on the TDWorld web site on 4/7/2014 by Mr. Ahmed Mousa.